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An estimate of the attraction domain with a specified exponential stability index in a wheeled robot control problem☆

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Abstract

The problem of synthesizing a law for the control of the plane motion of a wheeled robot is investigated. The rear wheels are the drive wheels and the front wheels are responsible for the turning of the platform. The aim of the control is to steer a target point to a specified trajectory and to stabilize the motion along it. The trajectory is assumed to be specified by a smooth curve. The actual curvature of the trajectory of the target point, which is related to the angle of rotation of the front wheels by a simple algebraic relation, is considered as the control. The control is subjected to bilateral constraints by virtue of the fact that the angle of rotation of the front wheels is limited. The attraction domain in the distance to trajectory - orientation space, is investigated for the proposed control law. Arrival at a trajectory with a specified exponential stability index is guaranteed in the case of initial conditions belonging to the given domain. An estimate of the attraction domain in the form of an ellipse is given.

1. Formulation of the problem

The motion of the wheeled robot is assumed to be planar. The symbol $X=(x, y)^T$ is used to denote a point of the plane, and the vectors are assumed to be column vectors. The target point is located at the middle of the rear axis of the platform of the robot and has the coordinates $X_c=(x_c, y_c)^T$. The orientation of the robot in the case of plane motion is defined by the single angle θ, formed by the central axis of the platform and the x axis. The platform of the robot has an actual angular rotation velocity $\dot{\theta}$ at each instant of time. We will denote the instantaneous positions of the ends of the wheel axles by X_i $(i=1, \ldots, 4)$. The condition for the motion of each of the four wheels without slipping means that the vectors of the instantaneous velocities V_i $(i=1, \ldots, 4)$ of the ends of the wheel axles are collinear with the planes of the wheels, and the normals to the each of these vectors intersect at a common point X_0. The point X_0, the position of which depends on time, is the instantaneous centre of the velocities of the robot platform. Moreover, the following relations must be satisfied

$$|\dot{\theta}| = \|V_1\|/\|X_1-X_0\| = \|V_2\|/\|X_2-X_0\| = \|V_3\|/\|X_3-X_0\| = \|V_4\|/\|X_4-X_0\| \tag{1.1}$$

The two rear wheels are driven and the two front wheels are responsible for the turning of the platform. If it moves translationally, then X_0 is located at an infinitely distant point and expression (1.1) gives a zero value for the angular velocity.

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In the case of motion without slipping, the relation $|\dot{\theta}| = ||V||/||X - X_0||$, which is similar to (1.1), is satisfied for any point X of the platform. At the same time, in the case of the points of the rear axis of the platform, the instantaneous centre of the velocities coincides with the instantaneous centre of curvature. In particular, for the target point, the quantity $||X_c - X_0||$ is the instantaneous radius of curvature of the trajectory of the point X_c and the quantity $1/||X_c - X_0||$ is the curvature of the trajectory which the target point describes. We will denote this quantity by u; then $||X_c - X_0|| = 1/u$.

Suppose L is the distance between the axes of the front and rear wheels and H is the distance between the front wheels. We then obtain the relations

$$\frac{uL}{1 - uH/L} = \operatorname{tg}\alpha_1, \quad \frac{uL}{1 + uH/2} = \operatorname{tg}\alpha_2 \tag{1.2}$$

which establish the relation between the curvature of the trajectory u at the target point and the angles of rotation α_1 and α_2 of the front wheels.

In order to achieve a given instantaneous curvature of the trajectory of the target point, the front wheel must be rotated through angles given by relations (1.2). The angles are measured counterclockwise. A positive (negative) value of the curvature u corresponds to the rotation to the the left (to the the right). The relation between the angles of rotation of the wheels and the curvature of the trajectory at the target point makes it possible to simplify the model and to choose the quantity u as a control.

We will put $v_c = ||V_c||$ in the case of forward motion and $v_c = -||V_c||$ in the case of reverse motion. The constraints on the angle of rotation of the wheels lead to the following bilateral constraints on the value of curvature of the trajectory:

$$-\bar{u} \le u \le \bar{u} \tag{1.3}$$

An expression for the value of $\bar{u}$ is obtained from the value of the maximum angle of rotation of the steering wheel in the desired direction. Taking account of condition (1.3), we can write the equations of motion of the robot in the form

$$\dot{x}_c = v_c\cos\theta, \quad \dot{y}_c = v_c\sin\theta, \quad \dot{\theta} = s_{\bar{u}}(u); \quad s_{\bar{u}}(u) = \begin{cases} -\bar{u} \text{ when } u \le -\bar{u} \\ u \text{ when } |u| < \bar{u} \\ \bar{u} \text{ when } u \ge \bar{u} \end{cases} \tag{1.4}$$

($s_{\bar{u}}(u)$ is a saturation function).

In many papers dealing with the control of wheeled robots (Refs. 1–5, etc.), the problem of synthesizing a control is solved, enabling the motion to be stabilized along a segment of a straight line or to be stabilized to a specified point of the plane. Here, the control can be continuous[3,4] or discontinuous.[5] Below, we consider the problem of synthesizing a control taking account of its boundedness. The boundedness of the control does not allow a guaranteed rate of decrease in the norm of the deviation from the specified trajectory to be attained for an arbitrary initial position of the target point and orientation of the robot platform. The problem of estimating the range of initial conditions, for which a synthesized control guarantees a specified rate of its exponential decay, is therefore of interest.

The trajectory is assumed to be a specified smooth curve

$$y = f(x) \tag{1.5}$$

with respect to which the following is assumed to be satisfied.

Assumption 1. The function *f*(*x*) is doubly continuously differentiable, and numbers $c_1 < 0$ and $0 < c_2 < \bar{u}$ exist such that

$$|f_x(x)| \le c_1, \quad |f_{xx}(x)| \le c_2, \quad \forall x \in R; \quad f_x = \partial f/\partial x, \quad f_{xx} = \partial^2 f/\partial x^2 \tag{1.6}$$

The condition $c_2 < \bar{u}$ cannot be omitted since it means that the maximum curvature of the target trajectory must not exceed the maximum curvature of rotation which is obtained for the maximum angle of rotation of the wheels.

2. Synthesis of the control for the motion of a mobile robot

Making the change of variables

$$\xi = x_c, \quad \eta = y_c - f(\xi), \quad \psi = \operatorname{tg}\theta - f_x(\xi)$$

system (1.4) can be reduced to the the system

$$\dot{\xi} = \upsilon_c \cos\theta, \quad \dot{\eta} = \upsilon_c \sin\theta - f_x(\xi)\upsilon_c \cos\theta, \quad \dot{\psi} = \frac{\upsilon_c s_{\bar{u}}(u)}{\cos^2\theta} - f_{xx}(\xi)\upsilon_c \cos\theta \tag{2.1}$$

In the new variables, the aim of the control is to ensure the conditions $\eta \to 0$ and $\psi \to 0$.

Assumption 2. The linear velocity of the platform $\upsilon_c(t)$ is positive and is non-zero

$$\upsilon_c(t) \geq \varepsilon_1 > 0 \tag{2.2}$$

Assumption 3. The relation

$$\cos\theta(t) \geq \varepsilon_2 > 0 \tag{2.3}$$

is satisfied on the trajectories of system (2.1).

Assumption 3 will subsequently be removed.

The variable η corresponds to the the deviation of the target point of the platform from the specified trajectory. In Eq. (2.1), we replace the derivative with respect to the time t by the derivative with respect to the the variable ξ (see Ref. 3). By virtue of the first of Eq. (2.1) and Assumptions 2 and 3, the variable ξ varies monotonically. A derivative with respect to the variable ξ is denoted by a prime, and we rewrite system (2.1) in the form

$$\xi' = 1, \quad \eta' = \psi, \quad \psi' = s_{\bar{u}}(u)A - f_{xx}(\xi); \quad A = [1 + (\psi + f_x(\xi))^2]^{3/2} \tag{2.4}$$

We now temporarily remove the bilateral constraints on the control in Eq. (2.4) and choose the control in the form

$$u = -\frac{2\lambda\psi + \lambda^2\eta - f_{xx}(\eta)}{A} \tag{2.5}$$

when $\lambda > 0$. At the same time,

$$\eta'' + 2\lambda\eta' + \lambda^2\eta = 0 \tag{2.6}$$

An exponential rate of decrease in the quantities η and ψ with an index $-\lambda$ follows from equality (2.6). However, a control of the form of (2.5) does not, generally speaking, satisfy bilateral constraints (1.4). On the other hand, the system

$$\xi' = 1, \quad \eta' = \psi, \quad \psi' = -s_{\bar{u}}\left(\frac{2\lambda\psi + \lambda^2\eta - f_{xx}(\xi)}{A}\right)A - f_{xx}(\xi) \tag{2.7}$$

with constraints on the control can not ensure an exponential rate of decrease in η and ψ with an index $-\lambda$.

We will put $z = (\eta, \psi)^T$.

Definition 1. The vector function $z(\xi)$ is said to the be a decreasing function with an exponential rate $-\mu$ when $\xi \geq 0$, if a quadratic function of the form

$$V(z) = z^T P z \tag{2.8}$$

exists, where the matrix $P > 0$, which satisfies the inequality

$$\frac{dV(z(\xi))}{d\xi} + 2\mu V(z(\xi)) \leq 0; \quad \xi \geq 0$$

We will investigate the properties of system (2.7). We put

$$C = (2\lambda, \lambda^2)^T \quad \text{and} \quad \sigma = 2\lambda\psi + \lambda^2\eta = C^T z$$

and rewrite the last equation of system (2.7) in the form

$$\psi' = -\Phi(\xi, \sigma); \quad \Phi(\xi, \sigma) = s_{\bar{u}}\left(\frac{\sigma - f_{xx}(\xi)}{A}\right)A + f_{xx}(\xi)$$

Note that

$$\Phi(\xi, \sigma) = s_{\bar{u}A}(\sigma - f_{xx}(\xi)) + f_{xx}(\xi)$$

We will now investigate the question of which initial conditions $z(0) = (\eta(0))$ and $\psi(0)^T$ ensure the exponential decrease of $z(\xi)$ in the trajectories of (2.7) at a rate of $-\mu$, where $0 < \mu \le \lambda$. In order to the estimate this domain of initial conditions, which we will denote by $\Omega(\mu)$, we shall consider a Lyapunov function of the form of (2.8) and seek an estimate of the domain $\Omega(\mu)$ in the form

$$\Omega_0(\alpha) = \{z\colon V(z) \le \alpha^2\}$$

We shall seek the matrix of the quadratic form p in the class of matrices satisfying the condition

$$P \ge I; \quad I = \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix} \tag{2.9}$$

We recall that the condition $A > B$ means that the matrix $A - B$ is positive definite. The other comparison operations have a similar meaning.

We will now investigate the question of for which values of μ and α the following inclusion is satisfied

$$\Omega_0(\alpha) \subseteq \Omega(\mu)$$

Lemma 1. *Let us assume that the condition*

$$z \in \Omega_0(\alpha) \tag{2.10}$$

is satisfied for a certain matrix P, which satisfies condition (2.9), *and a number* $\alpha > 0$

The following conditions are then satisfied:

$$-\sigma_0 \le \sigma \le \sigma_0; \quad \sigma_0 = \alpha\sqrt{C^T P^{-1} C} \tag{2.11}$$

$$s_{\bar{u} - c_2}(\sigma) \le \Phi(\xi; \sigma) \le s_{\bar{u}\bar{A} + c_2}(\sigma); \quad \bar{A} = (1 + (\alpha + c_1)^2)^{3/2} \tag{2.12}$$

Proof. It follows from conditions (2.9) and (2.10) that $\psi^2 \le \alpha^2$ whence, taking into account relations (1.8) and (2.4), we obtain condition (2.12). We will next consider the problem of the convex optimization $\sigma \to \max$ in the case of constraint (2.10). The necessary and sufficient conditions for an extremum have the form

$$2\nu P\begin{Vmatrix} \eta \\ \psi \end{Vmatrix} = C \tag{2.13}$$

where $\nu > 0$ is a Lagrange multiplier. On multiplying the last equation on the left by z^T and taking account of the fact that the extremum in the given case is reached on the boundary of the domain (2.10), we obtain

$$\nu\alpha^2 = \sigma_0/2$$

where σ_0 is the solution of the optimization problem. Multiplying Eq. (2.13) by $C^T P^{-1}$, we obtain

$$\nu\sigma_0 = C^T P^{-1} C/2$$

Combining the last two equalities, we have $\sigma_0 = \pm\alpha\sqrt{C^T P^{-1} C}$. Hence, at the point of the maximum,

$$\sigma = \sigma_0 = \alpha\sqrt{C^T P^{-1} C}$$

By considering the problem for a minimum and reasoning in a similar manner, we obtain that, at the point of the minimum $\sigma = \sigma_0$, whence condition (2.11) follows.

The first equation of system (2.7) guarantees a linear increase in the variable ξ and is not connected with the remaining two equations. The last two equations can be rewritten in the form

$$\eta' = \psi, \quad \psi' = -\Phi(\xi, \sigma) \tag{2.14}$$

Together with the function $\Phi(\xi, \sigma)$, we now introduce into consideration the function

$$\phi(\xi, \sigma) = \beta(\xi)\sigma \tag{2.15}$$

where

$$k_0 \le \beta(\xi) \le 1, \quad k_0 = \min\{(\bar{u} - c_2)/\sigma_0, 1\} \tag{2.16}$$

According to Assumption 1, we have $k_0 > 0$. It follows from Lemma 1 that the graph of the function $\Phi(\xi, \sigma)$ belongs to the sector which is bounded by the straight lines $\phi = k_0\sigma$ and $\phi = \sigma$. Plots of function (2.15) for all possible values of ξ belong to the same sector.

We will now extend the class of systems (2.14), by considering, alongside them, systems of the form

$$\eta' = \psi, \quad \psi' = -\phi(\xi, \sigma) \tag{2.17}$$

We require that the function $\beta(\xi)$ should satisfy the conditions for an absolutely continuous solution of system (2.17) to exist. If system (2.17) is to possess the property

$$V'(z) + 2\mu V(z) \le 0 \tag{2.18}$$

for any functions $\phi(\xi, \sigma)$ of the form of (2.15), (2.16), then property (2.18) will also be satisfied on the trajectories of system (2.14). In order to verify that property (2.18) is satisfied on the trajectories of system (2.17) for all possible functions $\phi(\xi, \sigma)$ of the form of (2.15), (2.16), we will use methods for analysing the absolute stability of control systems with time-varying non-linearity (see Refs. 6–9, p. 233).

Bearing in mind that the function $\beta(\xi)$ in expression 92.16) can take constant values in the range $k_0 \le \beta \le 1$, we will introduce into consideration the matrices of linear systems corresponding to all "intermediate" linear characteristics $\beta\sigma$:

$$A_\beta = \begin{Vmatrix} 0 & 1 \\ -\beta\lambda^2 & -2\beta\lambda \end{Vmatrix}$$

The linear matrix inequality

$$PA_1 + A_1^T P + 2\lambda P \le 0$$

is solvable, since the matrix A_1 has a multiple eigenvalue equal to $-\lambda$. □

Theorem 1. *Let us assume that the following linear matrix inequalities*

$$PA_\beta + A_\beta^T P + 2\mu P \le 0, \quad PA_1 + A_1^T P + 2\mu P \le 0, \tag{2.19}$$

$$\begin{Vmatrix} P & C \\ C^T & (\bar{u} - c_2)^2/(\alpha\beta)^2 \end{Vmatrix} > 0, \quad P \ge I \tag{2.20}$$

are solvable for the matrix P for certain values of $\alpha > 0$, $\mu > 0$ and $0 < \beta \le 1$.

Then, $\Omega_0(\alpha)$ is the domain of attraction of system (2.14) and the following inclusion is satisfied

$$\Omega_0(\alpha) \subseteq \Omega(\mu)$$

Proof. The stability of the zero solution of system (2.14) in the case of initial conditions satisfying condition (2.10) follows from the stability of the zero solution of system (2.17) for all possible functions $\phi(\xi, \sigma)$ of the form of (2.15). Moreover, the existence of a Lyapunov function, which satisfies condition (2.18), guarantees the exponential decrease of the variables z with an index $-\mu$, that is, $z \in \Omega(\mu)$. In order to satisfy condition (2.18) for all functions $\beta(\xi)$ which satisfy condition (2.16), it is necessary and sufficient that the following conditions be satisfied

$$PA_{k_0} + A_{k_0}^T P + 2\mu P \le 0, \quad PA_1 + A_1^T P + 2\mu P \le 0 \tag{2.21}$$

The value of k_0 is determined by the second formula of (2.16). Conditions (2.21) are satisfied for a certain matrix $P > 0$ for a stated value of k_0 if the linear matrix inequalities (2.19) are solvable for $P > 0$ for a certain β value, $0 < \beta \le k_0$.

By virtue of the last relation of (2.11) and condition (2.16), the condition $0 < \beta \le k_0$ means that

$$\beta^2 \le \frac{(\bar{u} - c_2)^2}{\sigma_0^2}, \quad \text{or} \quad c^T P^{-1} c \le \frac{(\bar{u} - c_2)^2}{\alpha^2 \beta^2}$$

Together with the condition $P > 0$, the last inequality implies that the matrix on the left-hand side of the first inequality of (2.20) is non-negative definite. Combining this assertion with the assertion of Lemma 1, we obtain that, when condition (2.10) and the conditions of Theorem 1 are satisfied, the solution of system (2.14), which begins at the point z, will be decreasing with an exponential rate $-\mu$. At the same time, condition (2.10) will be satisfied over the whole of the trajectory of system (2.14).

The simple assertion, following from Theorem 1, holds. □

Corollary. *The second condition of (2.20) implies that the domain $\Omega_0(\alpha)$ is inscribed in a circle of radius α. Since the required domain $\Omega_0(\alpha)$ is invariant for the trajectories of system (2.14), the trajectories of the system also do not leave a circle of radius α. In particular, the inequalities*

$$|\eta| \le \alpha, \quad |\psi| \le \alpha$$

are satisfied on trajectories beginning in the domain $\Omega_0(\alpha)$.

Suppose $z_0 = (\eta_0, \psi_0)^T$ is an initial datum. We now pose the question as to whether the point z_0 belongs to the domain $\Omega(\mu)$. The inequality

$$z_0^T P z_0 \le \alpha^2 \tag{2.22}$$

is linear in the coefficients of the matrix P. Verification that the conditions of Theorem 1 together with condition (2.22) are satisfied provides an answer to this question. Conditions (2.19), (2.20) and (2.22) represent a system of linear matrix inequalities. Numerical methods for checking the solvability of such systems have been thoroughly studied (see Ref. 8) and are included in various software packages.

We will now describe the computational scheme which is used in the practical verification of the conditions of Theorem 1. The first constraint of (2.20) is initially discarded and the problem of minimizing the trace of the matrix P is solved, subject to constraints (2.19) and the second constraint of (2.20). The parameter α does not appear in these constraints. The minimum value of $\beta \in (0, 1]$, for which constraints (2.19) and the second constraint of (2.20) of the problem remain compatible, is determined. Suppose P^* is the corresponding solution of these inequalities in the case of the limiting value β^*. It is immediately clear that, by virtue of the above-mentioned constraints, the minimum of the trace of the matrix P is reached in the case of the matrix P^*, which has a unique eigenvalue. In other words, the domain $\Omega_0(\alpha)$, which is inscribed in a circle of radius α, will be in contact with this circle for any value of α.

The maximum value of the parameter α^*, for which the first inequality of (2.20) is satisfied (it is necessarily satisfied for sufficiently small values of α) is then found for unchanged values of P^* and β^*. Strictly speaking, the maximum is found for those values of α for which the first inequality of (2.20) is satisfied as a non-strict inequality. In practical calculations, the final accuracy ϵ_0 is specified and the right-hand sides of the strict constraints-inequalities are replaced by $\epsilon_0 I$.

When the conditions of Theorem 1 are satisfied, the set $\Omega_0(\alpha)$ is invariant since the function $V(z(\xi))$ decays exponentially. It follows from this that, in the trajectories of system (2.7), the quantity $\psi(\xi)^2 = (\mathrm{tg}\,\theta(\xi) - f_x(\xi))^2$ is bounded by the quantity α^2 and, therefore,

$$|\mathrm{tg}\,\theta(\xi) - f_x(\xi)| \le \alpha$$

and, by virtue of Assumption 1,

$$|\mathrm{tg}\,\theta(\xi)| \le c_1 + \alpha$$

Consequently, $\cos\theta(\xi)$ does not change sign and, if the condition

$$|\cos\theta(0)| \ge (1 + (c_1 + \alpha)^2)^{-1/2}$$

is satisfied at the initial instant, it will be satisfied for all $\xi \ge 0$. The condition of Assumption 3 is therefore satisfied.

The assumption that $|f_x| \le c_1$ is also not restrictive. In fact, we assume that, having started to move from a point where this condition is satisfied, the robot was shifted to the a point x_1, y_1 where this condition is violated for the first time: $|f_x(x_1)| = c_1$. We now make the change of variables

$$\bar{x} = x\cos\theta_c + y\sin\theta_c, \quad \bar{y} = -x\sin\theta_c + y\cos\theta_c, \quad \bar{\theta} = \theta - \theta_c$$

where $\theta_c \in [-\pi/2, \pi/2]$ is an angle such that $\mathrm{tg}\,\theta_c = c_1$. In the new variables $\bar{x}$ and $\bar{y}$, the equations of motion (1.4) retain their form and the following condition is satisfied

$$|f_x(\bar{x}_1)| = 0; \quad \bar{x}_1 = x_1\cos\theta_c + y_1\sin\theta_c$$

If it is assumed that the transients terminate at the point x_1, y_1 and the condition $\psi_1 \approx 0$ is satisfied, then $\bar{\theta}_1 = \theta_1 - \theta_c \approx 0$ and, therefore, Assumption 3 is not violated after the change of variables. By virtue of the continuous differentiability of the function $f(x)$, Assumptions 1 and 3 are satisfied after the change of variables over a certain finite time.

3. Computational experiments

Computational experiments using the LMI Lab Matlab software package were carried out for the case $f(x) = \sin(2\pi x/50)$, $x \in [0, 100]$ when $\bar{u} = 0.1$, $\lambda = 2$, $v_c = 1$.

The left-hand part of Fig. 1 illustrates the difference between the trajectories of the motion of the target point in x and y coordinates taking into account ($\bar{u} = 0.1$, the dashed curve) and ignoring ($\bar{u} = \infty$, the solid curve) of the bilateral constraints on the control. The trajectory begins at the point (0, 1) with an initial orientation $\theta_0 = -\pi/4$. The different rate of convergence reflects the fact that the corresponding initial datum

$$\eta_0 = y_0 - f(x_0), \quad \psi_0 = \mathrm{tg}(\theta_0)$$

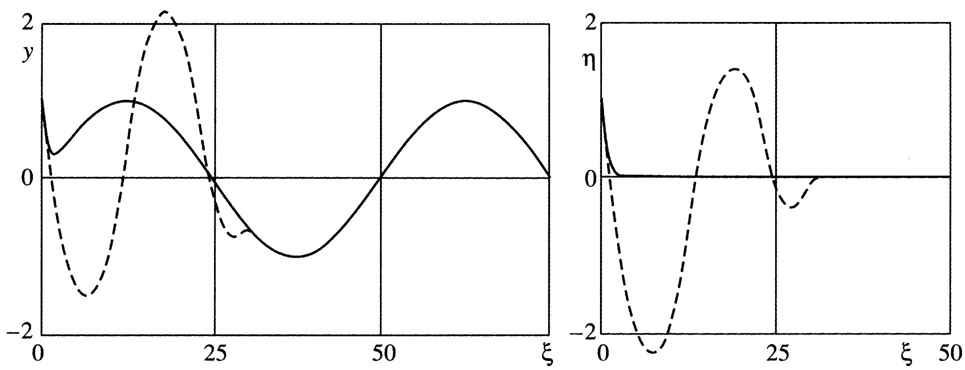

Fig. 1.

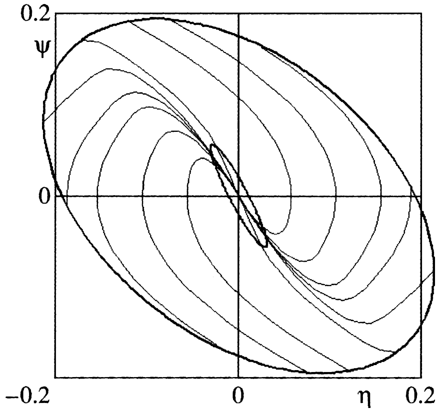

Fig. 2.

belongs to the the domain $\Omega(\mu)$ when $\mu < \lambda$. Graphs of the deviations $\eta(\xi)$ for the two above-mentioned cases are shown in the right-hand part of Fig. 1.

The simple case when $f(x) = 0$ is considered in order to illustrate the construction of attraction domains. As previously, $\lambda = 2$ and $\bar{\mu} = 0.1$. Estimates of the attraction domains, constructed for the cases when $\mu = 0.01$ (the outer ellipse) and $\mu = 1.6$ (the inner ellipse), are shown in Fig. 2. The trajectories of system (2.7), constructed for different initial data are also shown. The outer ellipse is defined by a quadratic form with the matrix $P^* = \left\| \begin{matrix} 1.61 & 0.762 \\ 0.762 & 1.95 \end{matrix} \right\|$ and the value $\alpha^* = 0.245$. The linear matrix inequalities in the formulation of Theorem 1, turned out to be solvable (and the matrix P^* is the solution) when $\beta^* = 0.1145$. In accordance with the corollary from Theorem 1, the quantities η and ψ do not exceed 0.245 in the trajectories which begin in the above-mentioned ellipse. The internal ellipse is determined by the parameters $P^* = \left\| \begin{matrix} 50.5 & 25.88 \\ 25.88 & 15.27 \end{matrix} \right\|$ and $\alpha^* = 0.08$ when $\beta^* = 0.84$.

It can be seen that both ellipses are invariant but the rates of decay of the corresponding quadratic forms in the trajectories of system (2.7) are different. Note that Theorem 1 only guarantees an estimate of the domains $\Omega(\mu)$.

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